Bell Work

$$X = \begin{bmatrix} 4 & -3 \\ 5 & 8 \end{bmatrix} \qquad Y = \begin{bmatrix} -6 & -1 \\ 2 & 3 \end{bmatrix}$$

1. $X + Y =$
2. $X \cdot Y =$
3. $Y \cdot X =$
4. How do you multiply matrices?

Multiplying Matrices:

$$A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix} \qquad B = \begin{bmatrix} -5 & 1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

2×3 matrix $\qquad$ 3×2 matrix

The number of the columns of the 1st matrix must be the same as the number of rows of the 2nd matrix.

The answer matrix will be 2×2.

Multiplying Matrices:

$$A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×3 matrix

$$B = \begin{bmatrix} -5 & 1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix}$$

2×2 matrix

$$D = \begin{bmatrix} 6 & -2 & 8 \end{bmatrix}$$

1×3 matrix

$$E = \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix}$$

3×1 matrix

Can we multiply AB? Yes, the answer matrix will be 2×2.

Can we multiply BA? Yes, the answer matrix will be 3×3.

Can we multiply AC? No.

Multiplying Matrices:

$$A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×3 matrix

$$B = \begin{bmatrix} -5 & 1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix}$$

2×2 matrix

$$D = \begin{bmatrix} 6 & -2 & 8 \end{bmatrix}$$

1×3 matrix

$$E = \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix}$$

3×1 matrix

Can we multiply CA? Yes, the answer matrix will be 2×3.

Can we multiply CD? No.

Can we multiply AE? Yes, the answer matrix will be 2×1.

Multiplying Matrices:

$$A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×3 matrix

$$B = \begin{bmatrix} -5 & 1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix}$$

2×2 matrix

$$D = \begin{bmatrix} 6 & -2 & 8 \end{bmatrix}$$

1×3 matrix

$$E = \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix}$$

3×1 matrix

Can we multiply *DE*? Yes, the answer matrix will be 1×1.

Can we multiply *ED*? Yes, the answer matrix will be 3×3.

Can we multiply *BD*? No.

Multiplying Matrices:

$$A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×3 matrix

$$B = \begin{bmatrix} -5 & 1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

AB would be a 2×2.

Multiplying Matrices:

$$A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×3 matrix

$$B = \begin{bmatrix} -5 & 1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

$$4(-5) + -6(3) + 3(0) = -38$$

$$AB = \begin{bmatrix} -38 & \\ & \end{bmatrix}$$

Multiplying Matrices:

$$A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×3 matrix

$$B = \begin{bmatrix} -5 & 1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

$$4(-5) + -6(3) + 3(0) = -38$$

$$4(1) + -6(2) + 3(-4) = -20$$

$$AB = \begin{bmatrix} -38 & -20 \\ & \end{bmatrix}$$

Multiplying Matrices:

$$A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×3 matrix

$$B = \begin{bmatrix} -5 & 1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

$$4(-5) + -6(3) + 3(0) = -38$$
$$4(1) + -6(2) + 3(-4) = -20$$
$$0(-5) + 1(3) + -7(0) = 3$$

$$AB = \begin{bmatrix} -38 & -20 \\ 3 & \end{bmatrix}$$

Multiplying Matrices:

$$A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×3 matrix

$$B = \begin{bmatrix} -5 & 1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

$$4(-5) + -6(3) + 3(0) = -38$$
$$4(1) + -6(2) + 3(-4) = -20$$
$$0(-5) + 1(3) + -7(0) = 3$$
$$0(1) + 1(2) + -7(-4) = 30$$

$$AB = \begin{bmatrix} -38 & -20 \\ 3 & 30 \end{bmatrix}$$

Multiplying Matrices:

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×2 matrix $\qquad$ 2×3 matrix

CA would be 2×3.

Multiplying Matrices:

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×2 matrix $\qquad 2 \times 3$ matrix

$$2(4) + -3(0) = 8$$

$$CA = \begin{bmatrix} 8 & & \\ & & \end{bmatrix}$$

Multiplying Matrices:

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×2 matrix $\qquad 2 \times 3$ matrix

$$2(4) + -3(0) = 8$$

$$2(-6) + -3(1) = -15$$

$$CA = \begin{bmatrix} 8 & -15 & \\ & & \end{bmatrix}$$

Multiplying Matrices:

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×2 matrix $\qquad 2 \times 3$ matrix

$$2(4) + -3(0) = 8$$

$$2(-6) + -3(1) = -15$$

$$2(3) + -3(-7) = 27$$

$$CA = \begin{bmatrix} 8 & -15 & 27 \\ & & \end{bmatrix}$$

Multiplying Matrices:

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×2 matrix $\qquad 2 \times 3$ matrix

$2(4) + -3(0) = 8 \qquad -1(4) + 9(0) = -4$

$2(-6) + -3(1) = -15$

$2(3) + -3(-7) = 27$

$$CA = \begin{bmatrix} 8 & -15 & 27 \\ -4 & & \end{bmatrix}$$

Multiplying Matrices:

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×2 matrix $\qquad 2 \times 3$ matrix

$2(4) + -3(0) = 8 \qquad -1(4) + 9(0) = -4$

$2(-6) + -3(1) = -15 \qquad -1(-6) + 9(1) = 15$

$2(3) + -3(-7) = 27$

$$CA = \begin{bmatrix} 8 & -15 & 27 \\ -4 & 15 & \end{bmatrix}$$

Multiplying Matrices:

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix} \qquad A = \begin{bmatrix} 4 & -6 & 3 \\ 0 & 1 & -7 \end{bmatrix}$$

2×2 matrix $\qquad 2 \times 3$ matrix

$$2(4) + -3(0) = 8 \qquad -1(4) + 9(0) = -4$$

$$2(-6) + -3(1) = -15 \qquad -1(-6) + 9(1) = 15$$

$$2(3) + -3(-7) = 27 \qquad -1(3) + 9(-7) = -66$$

$$CA = \begin{bmatrix} 8 & -15 & 27 \\ -4 & 15 & -66 \end{bmatrix}$$

Multiplying Matrices:

$$D = \begin{bmatrix} 6 & -2 & 8 \end{bmatrix}$$

1×3 matrix

$$B = \begin{bmatrix} -5 & -1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

DB would be 1×2.

Multiplying Matrices:

$$D = \begin{bmatrix} 6 & -2 & 8 \end{bmatrix}$$

1×3 matrix

$$B = \begin{bmatrix} -5 & -1 \\ 3 & 2 \\ 0 & -4 \end{bmatrix}$$

3×2 matrix

$$6(-5) + -2(3) + 8(0) = -36$$

$$6(-1) + -2(2) + 8(-4) = -42$$

$$DB = \begin{bmatrix} -36 & -42 \end{bmatrix}$$

Multiplying Matrices:

$$D = \begin{bmatrix} 6 & -2 & 8 \end{bmatrix}$$

1×3 matrix

$$E = \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix}$$

3×1 matrix

DE would be 1×1.

Multiplying Matrices:

$$D = \begin{bmatrix} 6 & -2 & 8 \end{bmatrix}$$

1×3 matrix

$$E = \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix}$$

3×1 matrix

$$6(-3) + -2(7) + 8(5) = 8$$

$$DE = \begin{bmatrix} 8 \end{bmatrix}$$

Multiplying Matrices:

$$E = \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix}$$

3×1 matrix

$$D = \begin{bmatrix} 6 & -2 & 8 \end{bmatrix}$$

1×3 matrix

ED would be 3×3.

Multiplying Matrices:

$$E = \begin{bmatrix} -3 \\ 7 \\ 5 \end{bmatrix}$$

3×1 matrix

$$D = \begin{bmatrix} 6 & -2 & 8 \end{bmatrix}$$

1×3 matrix

$-3(6) = 18 \qquad -3(-2) = 6 \qquad -3(8) = -24$

$7(6) = 42 \qquad 7(-2) = -14 \qquad 7(8) = 56$

$5(6) = 30 \qquad 5(-2) = -10 \qquad 5(8) = 40$

$$ED = \begin{bmatrix} -18 & 6 & -24 \\ 42 & -14 & 56 \\ 30 & -10 & 40 \end{bmatrix}$$

Multiplying Matrices:

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix}$$

2×2 matrix

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix}$$

2×2 matrix

$C^2 = C \times C$

C^2 would be a 2×2.

Multiplying Matrices:

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix}$$

2×2 matrix

$$C = \begin{bmatrix} 2 & -3 \\ -1 & 9 \end{bmatrix}$$

2×2 matrix

$2(2) + -3(-1) = 7$ $\quad 2(-3) + -3(9) = -33$

$-1(2) + 9(-1) = -11$ $\quad -1(-3) + 9(9) = 84$

$$C^2 = \begin{bmatrix} 7 & -33 \\ -11 & 84 \end{bmatrix}$$

Squaring a matrix only works on square matrices (1×1, 2×2, 3×3, etc.).
It doesn't work on 4×1, 3×2, etc., matrices.

Assignment:

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Tell whether each product is defined. If so, give its dimensions.

19. $A_{2\times 1}$ and $B_{2\times 3}$; AB

20. $A_{2\times 1}$ and $B_{2\times 3}$; BA

21. $C_{3\times 5}$ and $D_{5\times 1}$; CD

22. $C_{3\times 5}$ and $D_{5\times 1}$; DC

23. $E_{7\times 7}$ and $F_{6\times 7}$; EF

24. $E_{7\times 7}$ and $F_{6\times 7}$; FE

Use the following matrices for Exercises 25–29. Find each product, if possible.

$$A=\begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} \quad B=\begin{bmatrix} -3 & 0 \\ 7 & -2 \\ 0 & 1 \end{bmatrix} \quad C=\begin{bmatrix} -2 & 3 & -4 \\ 1 & -1 & 1 \\ 4 & 1 & 3 \end{bmatrix} \quad I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

25. AB

26. CA

27. CB

Use the following matrices for Exercises 31–40. Simplify, if possible.

$$Q=\begin{bmatrix} 4 & 13 & -9 \end{bmatrix} \quad S=\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix} \quad T=\begin{bmatrix} 2 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix} \quad A=\begin{bmatrix} 0 & -1 \\ -1 & 4 \\ 2 & 3 \end{bmatrix} \quad B=\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \end{bmatrix} \quad C=\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

31. S^2

32. B^2

33. T^2